

The Influence of Students' Commognition in Solving Integral Problems

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THE INFLUENCE OF STUDENTS' COMMUNICATIVE ABILITY IN SOLVING INTEGRAL PROBLEMS

Abstract: This study investigates the influence of students' communicative ability on their ability to solve integral problems, drawing from Sfard's communicative framework, which integrates cognition (intrapersonal thought) and communication (interpersonal discourse) as unified processes. Conducted among 60 undergraduate Mathematics Education students in Indonesia, the research utilized validated Mathematics Proficiency Test and Integral Problems tests, alongside semi-structured interviews, to assess performance across four communicative indicators: keywords, visual mediators, endorsed narratives, and routines. High-performing students excelled in all indicators, achieving 87.2% of the maximum score, reflecting a strong grasp of conceptual understanding and procedural fluency. Conversely, medium- and low-performing students demonstrated significant challenges, particularly with visual mediators and endorsed narratives, often struggling to construct and utilize mathematical representations and theorems effectively. Statistical analyses revealed a significant correlation between communicative ability and problem-solving ability (t-test $p < 0.05$; F-test $p < 0.05$). These findings underscore the critical role of structured pedagogical strategies, including explicit instruction in mathematical communication and visualization, to bridge gaps in mathematical discourse. The study highlights the importance of collaborative learning environments and the integration of communicative principles in curriculum design to enhance mathematics education and equip students for advanced problem-solving tasks. Further research is recommended to evaluate long-term impacts and the potential of technology-based interventions in fostering communicative skills.

Keywords: Communicative ability, students, integral problems

Introduction

Sfard's perspective is known as the "communicative framework", with the term "communicative ability" emphasizing the inseparable relationship of "cognition" and "communication" (Sfard, 2008). The term communicative ability which includes thinking (individual cognition) and communication (interpersonal), as a combination of the words cognition and communication, emphasizes the fact that these two processes are different manifestations (intrapersonal and interpersonal) of the same phenomenon. "thinking is defined as the individualized version of interpersonal communication - as a communicative interaction in which one person plays the roles of all interlocutors" which means that thinking is defined as the individualized version of interpersonal communication (Sfard, 2008). As interactive communication where one person plays the role of all interlocutors. Specifically, cognition is an interpersonal expression while communication is an interpersonal expression of a phenomenon. So communicative ability is a unity of development from the process of thinking and communicating (Benedictus *et al.*, 2015).

According to Vygotsky (1978), knowledge, concepts, and higher mental functions are the result of culture. Constantly, these different parts are modified because of the collective efforts of humans. Vygotsky views language as an instrument for developing thought. It is the similarity of Wittgenstein and Vygotsky's theories that show that learning mathematics can be facilitated by the meaning and language in the discourse of society. Learning is not located in the head or outside the individual, it is the result of the relationship between the individual and the social world (Danoebroto, 2015).

Communication is an important element in every learning (Wichelt & Kearney, 2009). In lecture activities, communication recognizes students' abilities, through a description of understanding the material being studied and can then provide assistance for student difficulties (Elias *et al.*, 2012).

Communication can help students develop their understanding of mathematics and sharpen their thinking skills (Kaur, 2011). When students are challenged to think and communicate the results of their thinking to others, it is often unclear and full of errors. Many students make mistakes in expressing terms and using words when explaining the definition of mathematics systematically. Inappropriate communication is thought to be due to differences in students' mindsets when they are in high school compared to when they are in college (Friedenberg & Silverman, 2006).

Based on the definition above, cognition and communication are activities that offer students the possibility to develop understanding in learning mathematics. Cognitive and communication factors play an important role in children's success in the learning process because most activities in learning are always related to understanding and problem solving. Understanding here is focused on understanding the concept of facts, concepts, principles and procedures which are aspects in the cognitive domain of learning objectives, because understanding concepts is also the development of mathematical knowledge that someone has (Manolio *et al.*, 2003).

Cognition is a general term that includes all models of understanding, namely perception, imagination, judgment and reasoning (Kuper & Kuper, 2000). Some opinions of communication are seen as a window to the thinking process, while in other traditions, communication is the same as thinking (Tabach & Nachlieli, 2016).

“Communication is a collectively performed patterned activity in which action A of an individual is followed by action B of another individual so that (a) A belongs to a certain well-defined repertoire of actions known as communicational. (b) Action B belongs to a repertoire of reactions that fit A, that is, action recurrently observed in conjunction with A” (Sfard, 2008).

Sfard (2008) said that in communication there is action-reaction. Action must be an act of communication that will be followed by an appropriate reaction. Action and reaction in practical action.

Cognition as communicating with oneself whose activities are carried out in groups (Sfard, 2008). Furthermore, Sfard defines cognition as a form of "individualized form of the activity of communicating", in the sense that cognition is not only a separate individual process with communication actions and does not have to be interpersonal. Based on the opinion above, the process of cognition and the process of interpersonal communication are forms of phenomena that are basically the same (Czajkowska *et al.*, 2010).

"Commognition is primarily a participationist theory, learning only takes place through the individual thoughtful participation in mathematical discourse", which can be interpreted that commognition is basically a participation theory because it only occurs through individual participation in mathematical discourse (Berger, 2013). According to Sfard (2008) "mathematics is a multilayered recursive structure of discourses-about-discourse", because mathematics is an autopoiesis which means it can create itself. Furthermore, Sfard explains "the different types of communication, and thus of commognition, that draw some individuals together while excluding some others will be called discourses", in other words the different types of communication are called discourses.

Based on the various opinions above, it can be concluded that commognition is a combination of thought (individual cognition) and communication (interpersonal) which can be conceptualized as communication between a person and themselves and in a group.

Commognition Indicator

Furthermore, the communication indicators according to Sfard (2008) are as follows:

Table 1: Commognition indicators

Commognition Indicators	Deskriptor
1. Keywords	Using terms in mathematics Using special words
2. Visual Mediators	Using mediators in the form of graphs, diagrams and symbols
3. Endorsed Narrative	Using definitions Using Theorems and proofs
4. Routines	Generalize, conclude or estimate the answers obtained
Eksplorations	Carrying out construction that produces a new supporting narrative Conduct substantiation to decide whether to support the previously constructed narrative. Recall to be able to recall previous narratives
Deed	Performing actions that are easy to do because of the patterned sequence of actions
Rituals	Performing socially oriented repetitive patterns

Theoretical Background

The theoretical foundation of this research is rooted in the "commognitive framework," a perspective pioneered by Sfard (2008) that bridges cognition and communication. The term "commognition" embodies the integration of thinking (intrapersonal cognition) and communicating (interpersonal communication), signifying that these are different manifestations of the same phenomenon. Sfard (2008) emphasizes that thinking is "an individualized version of interpersonal communication," where an individual acts as both speaker and listener in a mental dialogue. This perspective positions commognition as a unified construct that enhances our understanding of cognitive and communicative processes in mathematical learning.

The theoretical underpinnings of this study also align with Vygotsky's (1978) sociocultural theory, which asserts that higher mental functions, including problem-solving and concept formation, are products of social interaction and cultural tools such as language. Vygotsky viewed language as a mediator for thought development, emphasizing that learning mathematics involves the interplay between individual cognition and societal discourse. Similarly, Wittgenstein's (1953) perspective on the role of language in shaping thought resonates with this framework, highlighting the communal nature of mathematical understanding.

Communication has been established as a critical component in mathematics education (Wichelt & Kearney, 2009). Effective communication allows students to articulate their understanding, refine their reasoning, and collaborate to solve complex problems. However, transitioning from secondary to tertiary education often reveals gaps in students' mathematical discourse, resulting in misconceptions and challenges in expressing mathematical concepts systematically (Kaur, 2011). Addressing these gaps requires fostering explicit and structured communication strategies.

Sfard (2008) delineates four key indicators of commognition that serve as analytical tools in examining mathematical discourse. These indicators include keywords, which involve the use of specific mathematical terms and specialized vocabulary; visual mediators, which include employing diagrams, graphs, and symbols as tools for mathematical reasoning; endorsed narratives, which involve developing and utilizing mathematical definitions, theorems, and proofs to substantiate reasoning; and routines, which refer to repeating structured processes, including generalization, conclusion-making, and exploration of new narratives. These elements highlight how students engage with mathematical problems and express their solutions.

Cognition, encompassing perception, imagination, judgment, and reasoning, serves as the foundation of mathematical thinking (Kuper & Kuper, 2000). Communication acts as both a window into the cognitive process and an integral part of it (Tabach & Nachlieli, 2016). This duality underscores the significance of fostering commognitive skills to support mathematical proficiency. Mathematical thinking benefits significantly from collaborative discourse, as it encourages peer learning and collective problem-solving.

The theoretical model of this study aligns with the participationist view, which posits that learning occurs through active engagement in mathematical discourse (Berger, 2013). According to Sfard (2008), mathematics is a recursive structure of discourses that evolve through iterative communication. These discourses shape the learners' ability to interact with and internalize mathematical concepts. This iterative process is critical in helping students transition from basic understanding to advanced mathematical reasoning.

The integration of commognition in problem-solving is particularly pertinent in addressing integral problems, as it necessitates the interplay of conceptual understanding and procedural fluency. Previous research highlights that the cognitive demand of mathematical tasks is amplified when students are required to communicate their reasoning explicitly (Stillman, 2006). Such integration fosters deeper learning and the ability to tackle increasingly complex problems, emphasizing the importance of both clarity in communication and depth in understanding.

In summary, the commognitive framework provides a robust theoretical lens for examining the intersection of cognition and communication in mathematical learning. By leveraging this framework, the present study seeks to explore the influence of students' commognition on their ability to solve integral problems, thereby contributing to the discourse on effective mathematics education and addressing gaps in instructional methodologies.

Method

This research involved a class consisting of 60 undergraduate Mathematics Education students of the State Islamic Universities in Makassar, Indonesia. The two research instruments are The Mathematics Proficiency Test consisting of 5 essay questions, and the Integral Problems Test consists of 6 essay questions. Both instruments are validated by three associate professors of mathematics education and the aspects of content and feasibility with scores of 3.81 and 3.90 (maximum 4.00), respectively. Based on the empirical test, the difficulty index for both instruments are as moderate with scores 0.698 and 0.70, respectively, and the differentiating power coefficient of 0.562 and 0.71, respectively. In addition, both instruments are valid with coefficients of 0.997 and 0.993 respectively. Employ the Cronbach's Alpha test and $\alpha=5\%$, both instruments are reliable with coefficients of 0.836 and 0.889 respectively. Instrument testing was carried out on ten students at a private university in Enrekang Regency, Indonesia. The results of the two instruments were categorized as moderate respectively with a difficulty index of 0.753 and 0.733, and a discriminating power of 0.624 and 0.751.

Both instruments are valid and suitable for use for data collection. Furthermore, 60 students worked on the two instruments on different days and sequentially with 100 minutes each for the Mathematics Ability Test and 120 minutes for the Integral Problems Test and were supervised directly by the researcher with the assistance of one lecturer who taught the course.

Four days after working on the Integral Problems Test questions, sixty students were interviewed based on the Integral Problems Test answer sheets of the sixty students. Semi-structured interviews were conducted to explore the achievement of four commognitive indicators, namely keywords, visual mediators, endorsed narrative, and routines.

Result and Discussion

Descriptive Statistical Analysis

Based on research conducted on the Undergraduate Study Program in Mathematics Education at the State Islamic Universities in Makassar, Indonesia, the following results were obtained:

The results of the Mathematics Proficiency Test (TKM) of students obtained the lowest score of 46, the highest score of 93, an average (mean) of 62.13 and a standard deviation of 10.21. From the results of the Mathematics Proficiency Test, 5 students with high mathematics ability (score ≥ 80), 30 students with medium mathematics proficiency (score 60-79) and 25 students with low mathematics proficiency (score ≤ 59) were also obtained. While the results of the Integral Problem Test (TPI) of students obtained the lowest score of 42, the highest score of 91, an average (mean) of 66.85 and a standard deviation of 9.35. From the results of the Integral Problem Test (TPI) 6 students were also categorized as high scores (score ≥ 80), 45 students were categorized as medium scores (score 60-79) and 9 students were categorized as low scores (score ≤ 59). From the results of the Integral Problem Test, it was also obtained that students with high mathematical abilities were able to work on integral problems based on the four existing cognitive indicators of 87.20% of the maximum score that should be obtained, medium of 68.11% and low of 51.33%.

Table 2: Student commognition results

No.	Commognition Indicator	f1	f2	f3	f4	f5	f6
1.	Keywords	44	50	45	46	51	11
2.	Visual Mediators	0	0	34	0	0	11
3.	Endorsed Narrative	38	55	48	53	47	0
4.	Routines	29	47	6	40	37	0

Keterangan

- f1 : The number of students who answered question number 1 correctly for the 4 communication indicators
- f2 : The number of students who answered question number 2 correctly for the 4 communication indicators
- f3 : The number of students who answered question number 3 correctly for the 4 communication indicators
- f4 : The number of students who answered question number 4 correctly for the 4 communication indicators
- f5 : The number of students who answered question number 5 correctly for the 4 communication indicators
- f6 : The number of students who answered question number 6 correctly for the 4 communication indicators

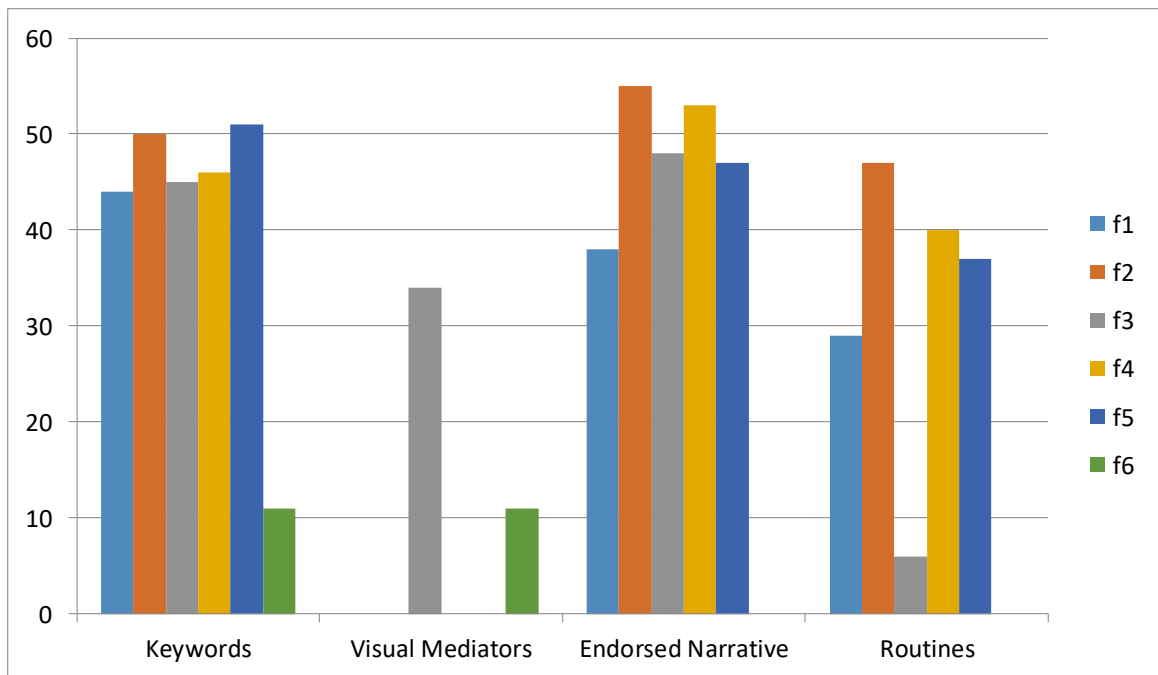


Figure 1: Achievement of commognition indicators

Data Analysis Requirements Test

Normality test

To calculate the data normality test with Chi-Square, the following formula is usually used:

$$x^2 = \sum \frac{(O_i - E_i)}{E_i}$$

Description:

O_i = Observation value

E_i = Expected value, class interval area based on normal table multiplied by N (total frequency) ($\pi \times N$)

N = The number of numbers in the data (total frequency)
(Sudijono, 2009)

In this study, SPSS Version 26 was used to test normality with the following results:

Table 3: Normality test

		Unstandardized Residual
N		60
Normal Parameters ^{a,b}	Mean	.0000000
	Std. Deviation	1.88165271
Most Extreme Differences	Absolute	.214
	Positive	.125
	Negative	-.214
Test Statistic		.214
Asymp. Sig. (2-tailed)		.064 ^c
a. Test distribution is Normal.		
b. Calculated from data.		
c. Lilliefors Significance Correction.		

Based on the calculation results with SPSS version 26, the significance value (sig.) is above 0.05 (sig. > 0.05) which is 0.064 so it can be concluded that the data is normally distributed, so the data analysis process can be carried out using parametric statistics.

Homogeneity Test

According to Nuryadi (2017) and Sugiyono (2019), the homogeneity test is a statistical test procedure designed to show that two or more sets of sample data from a population have the same variance. As a basis for making decisions on the homogeneity test are: (1.) If the possible sig. value <0.05 then the variance of two or more population groups or data samples is not homogeneous. (2.) If the possible sig. value >0.05 then the variance of two or more population groups or data samples is homogeneous. Manual calculation of the homogeneity test according to Sugiyono (2010), if the data is normal, the analysis of variance requires a homogeneity test of the variance using the F test.

$$F = \frac{\text{Largest Variance}}{\text{Smallest Variance}}$$

In this study, the homogeneity test used SPSS version 26 with the following results:

Table 4: Homogeneity test

	Levene Statistic	df1	df2	Sig.
Based on Mean	.737	2	177	.480
Based on Median	.532	2	177	.589
Based on Median and with adjusted df	.532	2	164.196	.589
Based on trimmed mean	.727	2	177	.485

Based on the calculation with SPSS above, we obtain a significance value (sig.) of 0.480; From these results, it can be concluded that the sig value for TKM, Komognisi and TPI is above 0.05 (sig > 0.05), so it can be concluded that the sample comes from a population that has homogeneous variance.

Hypothesis Test

The hypothesis used is as follows:

H_0 : There is no significant influence of student commognition in solving integral problems.

H_1 : There is a significant influence of student commognition in solving integral problems.

To test the above hypothesis we use the t-test and F-test using SPSS Version 26, where the rules are as follows:

For the t-test and F-test If significance (sig.) <0.05 then H_0 is rejected, H_1 is accepted.

Table 5: F-test

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	4944.754	2	2472.377	674.619	.000 ^b
	Residual	208.896	57	3.665		
	Total	5153.650	59			

a. Dependent Variable: TPI

b. Predictors: (Constant), Komognisi, TKM

Table 6: t-test

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	11.152	1.536		7.259	.000
	TKM	.922	.054	1.007	16.952	.000
	Komognisi	-.026	.050	-.031	-.523	.013

a. Dependent Variable: TPI

Based on the calculation results with SPSS Version 26 above, using the t test, a significance value (sig.) of 0.013 (sig. <0.05) was obtained and using the F test, a significance value (sig.) of 0.00 (sig. <0.05) was obtained so that H_0 was rejected, H_1 was accepted, so it can be concluded that there is a significant influence of student commognition in solving integral problems.

Next, after conducting interviews with 60 students, when asked from question number 1 to question number 6, which question number was the most difficult for you to solve?, All 60 students answered questions number 3 and number 6. Furthermore, when the question was continued by asking for questions number 3 and number 6, which part was difficult for you to do?, Of the 60 students answered the part of making graphs/charts based on what was known from the question. What's next?, They answered next to calculate the volume of the rotating object because they did not know or forgot the formula used.

From the results of this interview, it was found that students still had difficulty in fulfilling the commognition indicator, namely the Visual mediators indicator, namely that students could not draw graphs/charts based on what was known from the question. Furthermore, from the interview, it was found that students still had difficulty in fulfilling the commognition indicator endorsed narratives, namely that students had difficulty in using definitions and using existing Theorems and proofs to work on integral problems.

Discussion

This study reveals the significant influence of commognition on students' ability to solve integral problems, as evidenced by statistical analysis. The t-test (significance value 0.013) and F-test (significance value 0.000) show a strong relationship between commognitive skills and problem-solving success. These findings align with Sfard's (2008) commognitive framework, emphasizing the importance of integrating cognition and communication in effective mathematics learning.

High performing students successfully met all four commognition indicators keywords, visual mediators, endorsed narratives, and routines achieving an average of 87.20% of the maximum score. Their ability to effectively utilize mathematical terminology, visual representations, and logical reasoning reflects mastery in both conceptual understanding and procedural fluency. This finding aligns with studies by Stillman (2006) and Cobb & Yackel (1996), which highlight the importance of mathematical discourse in fostering higher order thinking skills.

However, medium and low performing students faced significant challenges in meeting all indicators, particularly in visual mediators. These difficulties are likely due to limited prior experience with tasks requiring the integration of visualization and logical reasoning, as noted by Schoenfeld (2016). This underscores the need for more structured pedagogical approaches to help students understand and integrate visual representations in problem-solving.

Another contributing factor is the gap in mathematical discourse between secondary and tertiary education. This gap, as noted by Kaur (2011), often results in students struggling to systematically communicate mathematical concepts. According to Mason (2012), addressing this gap requires explicit teaching of mathematical language and structured communication strategies.

Furthermore, the findings highlight the importance of collaborative learning environments in developing commognitive skills. Peer interactions allow students to articulate their understanding, discuss errors, and refine their logical reasoning. Goos *et al.* (2002) demonstrated that cooperative problem solving enhances cognitive engagement and reasoning, which is particularly beneficial for medium and low performing students.

From a theoretical perspective, this study reinforces the participatory learning model, which views mathematics learning as participation in discourse (Sfard, 2008). Lampert (1990) also emphasized that mathematical dialogue not only deepens conceptual understanding but also builds more effective problem solving strategies. This underscores the importance of integrating discourse into mathematics curriculum design.

Another implication is the integration of tasks emphasizing explicit mathematical communication. As noted by Boaler (2016), tasks designed to enhance mathematical communication skills can foster creativity and resilience in solving complex problems. This strategy can be applied across contexts to support more inclusive learning.

In addition, this study highlights the need for further research to explore the long-term effects of commognition based teaching. Research by Silverman *et al.* (2021) found that sustained exposure to communication based mathematics tasks improves students' confidence in problem-solving and adaptability to new challenges. This approach could be used to bridge learning gaps among students with diverse backgrounds.

This study also suggests the importance of cultural factors in shaping students' commognitive practices. According to Clements & Sarama (2020), educational approaches that consider cultural contexts can help students better understand and apply mathematical concepts. This provides opportunities to develop pedagogical strategies that better meet the needs of students from diverse backgrounds.

Overall, these findings affirm the importance of balanced teaching that integrates procedural fluency and conceptual understanding. By incorporating explicit instruction in commognition, opportunities for collaborative learning, and culturally inclusive approaches, educators can help students prepare for the challenges of advanced mathematics. These implications are relevant to improving the effectiveness of mathematics education at all levels.

Conclusion

This study demonstrates that commognition significantly influences students' ability to solve integral problems. High performing students exhibit the capability to fulfill all four commognition indicators: keywords, visual mediators, endorsed narratives, and routines. This reflects strong mastery in conceptual understanding and procedural fluency, which are essential foundations for effective mathematics learning. However, medium and low performing students face challenges in meeting all indicators, particularly in using visual mediators and endorsed narratives. These gaps highlight the need for structured teaching approaches, such as explicit instruction on visual representation and mathematical communication. Additionally, collaborative learning and continuous feedback can help students gradually improve their understanding and enhance their problem solving skills. These findings underline the importance of integrating commognitive skills into mathematics education. By adopting balanced learning strategies that emphasize both procedural fluency and conceptual understanding, educators can better prepare students for the challenges of advanced mathematical tasks and their real world applications. Further research is recommended to explore the long term effects of commognition based interventions and the role of technology in supporting the development of these skills.

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