Proceedings of the International Conference on Education, Teaching, and Learning, Vol. 3, 2024, pp. 72-80

ISSN 2820-2155
DOI: https://doi.org/10.32789/edcon.2024.3107

Strategic Flexibility of Pre-Service Mathematics Teachers in Problem Solving: A Literature Review

R.Ramadhona[1,2], Manuharawati[1], A.Wintarti[1]

[1]*Mathematics Education Study Program, Universitas Negeri Surabaya, Indonesia.*
rezky.21031@mhs.unesa.ac.id, manuharawati@unesa.ac.id, atikwintarti@unesa.ac.id
[2]*Mathematics Education Study Program, Universitas Maritim Raja Ali Haji, Indonesia.*
rezky.21031@mhs.unesa.ac.id

STRATEGIC FLEXIBILITY OF PRE-SERVICE MATHEMATICS TEACHERS IN PROBLEM SOLVING: A LITERATURE REVIEW

Abstract: A mathematical problem-solving strategy involves understanding and analyzing information, then translating it into mathematical form to find solutions. Strategic flexibility is the ability to use various strategies and being able to change and adapt strategies according to the situation at hand. This study presents a literature review on the strategies employed and strategic flexibility pre-service mathematics teachers in problem-solving. The research uses the Systematic Literature Review method, which consists of three stages, namely planning, conducting, and reporting. The articles were searched in the Publish or Perish database. The literature review reveals that pre-service mathematics teachers use three main types of strategies for solving mathematical problems: cognitive strategies, process-oriented strategies, and solution-focused strategies. The strategies used in solving problems can be algebraic strategies and non-algebraic strategies. Meanwhile, strategic flexibility consists of three criteria: strategy adaptation, intra-task strategic flexibility, and inter-task strategic flexibility. The strategic flexibility of pre-service teachers needs to be improved, pre-service teachers must have knowledge of strategy and have good flexibility in learning.
Keywords: Pre-service mathematics teachers, problem-solving, strategic flexibility

Introduction

A strategy assists individuals in solving problems. Strategy is a series of logical actions, such as analyzing information, building problem representations, selecting tools for solutions, and planning a series of steps to reach a solution. The strategy chosen depends on the type of problem at hand. Different individuals may approach the same problem with varied strategies, and even the same person might use different strategies for similar problems (Fazio, 2016). It can be seen that the strategies for solving problems vary, depending on the person and the problem at hand.

Mathematical problem solving is a process that students go through in solving mathematical problems starting from the beginning of getting the problem until the end of the problem is solved using all the knowledge they have (Polya, 1981). Problem-solving requires students to apply and integrate many mathematical concepts and skills and make decisions (Tambychik & Meerah, 2010). Many research results report that students have difficulty solving mathematical problems. It is due to the lack of mathematical skills possessed by students.

(Kilpatrick *et al*., 2001) express mathematical skills or mathematical proficiency as an aspect that summarizes what students should master to succeed in mathematics. (Kilpatrick *et al*., 2001) divide mathematical proficiency into five interrelated competencies: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

Strategic competencies that are part of mathematical proficiency have an essential role in solving mathematical problems and overcoming problems in real life (Awofala, 2017). Strategic competence is a person's ability to act to solve problems. So, strategic competence is needed to determine the direction of problem-solving and to understand which stages must be followed to reach a solution (Egodawatte & Stoilescu, 2015). (Kilpatrick *et al*., 2001) said strategic competence is the ability to understand the problem and formulate it, then represent the problem in various ways to get a solution to the problem. Similarly, (Ostler, 2011) describes strategic competence as the ability to construct an appropriate mathematical model and identify the correct strategy to solve a problem.

The teacher's strategic competence includes the ability to (a) formulate, represent, and solve problems; (b) model mathematical ideas; and (c) demonstrate representational fluency, i.e., the ability to translate and relate within and between various representations accurately, efficiently, and flexibly (J. M. Suh, 2014). Teachers' knowledge of mathematics influences how they are taught later (Almeida & Bruno,

2014). Teachers should have more information about problem-solving strategies so that they can teach these strategies to their students (Celebioglu *et al*., 2010). Hence, a teacher should have a variety of mathematical problem-solving strategies. Therefore, it is necessary to conduct a literature review to identify how strategic flexibility of pre-service mathematics teachers in solving problems.

In this study, Research Questions are:

a) What strategies do pre-service mathematics teachers use in problem-solving?
b) How is the strategic flexibility of pre-service mathematics teachers in problem solving?

Method

This research uses the method of Systematic Literature Review (S.L.R.). The S.L.R. method comprises three stages: planning, conducting, and reporting, as illustrated in Figure 1.

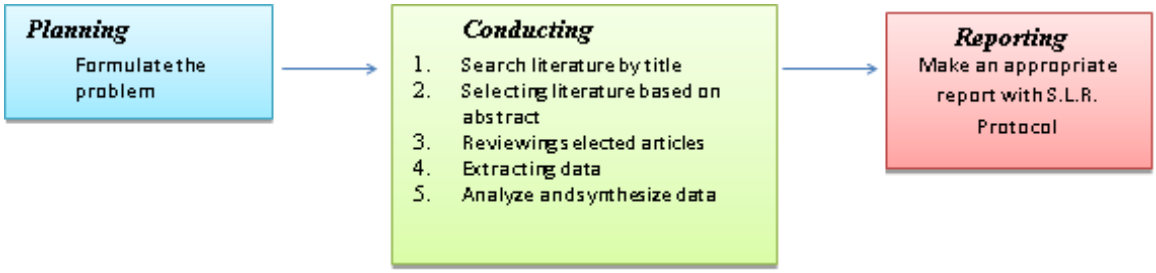

Figure 1: S.L.R. stages

At the stage of conducting, the article was searched in the database Publish or Perish with the title keyword "strategic competence", and found 147 articles; for the keyword "Strategic flexibility," as many as 200 articles, and keywords "problem-solving strategies" found as many as 200 articles. Furthermore, articles were selected based on abstracts. Hence 19 articles were selected that were relevant. Furthermore, the analysis and synthesis of the article were carried out

Result and Discussion

Based on the results of the literature selection, 19 relevant articles were obtained and were further reviewed. Articles that are used as references regarding problem-solving strategies are grouped based on the main topics studied. The Topics are:

a) Strategy Use
b) The research related to the strategies used by students in solving problems is research conducted by (Avcu & Avcu, 2010), (Celebioglu *et al*., 2010), (Jiang & Chua, 2010), (J. Suh *et al*., 2012), (Almeida & Bruno, 2014), (Fazio, 2016), (Saygılı, 2017), (Öçal *et al*., 2019), (Rubenstein *et al*., 2019), (KOCA & GÜRBÜZ, 2021).
c) Strategic Flexibility
d) Research that discusses the flexibility of students' strategies in solving problems (Arslan & Yazgan, 2015), (Keleş & Yazgan, 2021), (Gavaz *et al*., 2021), (Keleş & Yazgan, 2022),(Friesen *et al*., 2022), (Segura & Ferrando, 2023)
e) Strategic competence, researched by (Egodawatte & Stoilescu, 2015), and (Copur-gencturk, 2021)
f) Strategy Diversity, researched by (Clements *et al*., 2019)

From the grouping of articles above, it can be seen the following:

a. Types of strategies used in problem-solving

The strategy of prospective teachers in solving problems needs to be improved. (Avcu, 2010) found that the strategy of prospective mathematics teachers in elementary schools in solving problems was still limited, such as making a drawing, accounting for all possibilities, adapting a different point of view, finding a pattern, organizing data, logical reasoning, and working backward. The research developed by (J.Suh *et al*., 2012) describes the teacher's problem-solving strategy in rational numbers, namely using a ratio table and counting, finding multipliers. Likewise, the results of research by Almeida & Bruno (2014) related to the teacher's strategy in solving simple addition problems involving negative numbers, namely use an operation involving positive numbers, use an operation involving negative numbers, use a number line, count sequentially, give a verbal explanation, make a drawing.

Everyone has a different strategy for solving problems. (Jiang & Chua, 2010) conducted research on Chinese and Singaporean students. Chinese students use traditional strategies such as arithmetic, and algebraic, and Singapore students use the strategy "Model and Unitary" Another study explains that elementary-level students more often use the strategy look for a pattern, even though they draw a diagram and simplify the problem is a strategy that is often used together in problem-solving (Celebioglu *et al*., 2010). Students employ various strategies for problem-solving, including systematic listing, guessing and checking, diagram drawing, working backward, reasoning, forming equations, pattern recognition, and problem simplification. The strategies that are often used are guessing and checking, and reasoning. Strategies that are rarely used are forming equations (Keleş & Yazgan, 2021).

The strategy used is not always an algebraic strategy. But non-algebraic strategies are also often used by students. (Öçal *et al*., 2019) said the most common non-algebraic strategies used in completing the calculation process are a)guess and check, a student makes predictions and verifies answers by stating the conditions of the problem, and each next guess is made based on the results obtained from previous predictions, b) work backward, problem solution starts from the end and each operation is reversed to reach the unknown start, c) make a drawing, the relationship between different pieces of information in a given problem is represented using a visual model, and d)manipulate the structure, the solution starts with rearranging the information given in the problem, then the problem is solved by making logical reasoning from the rearrangement. Students who fail to choose the right strategy often fail to demonstrate proper algebraic thinking in the problem-solving process. Hence, to be able to choose the right strategy, students need to have an overall view of the problem, not just focus on a few parts (Egodawatte & Stoilescu, 2015).

Based on the results of the analysis of the article above, it can be seen that the strategies used by prospective mathematics teachers in solving problems vary. This is consistent with the diversity of strategies grouped by (Clements *et al*., 2019) into three:

1. Strategic cognitive is a procedure directed at the goals and efforts that students use to help solve problems
2. Strategic processes is a form of procedural knowledge where a child knows how to do something so as to improve their ability to solve problems
3. Strategic solution is the final form of problem-solving.

The indicators to make it easier to see the strategies used in solving mathematical problems are explained by (Copur-gencturk, 2021), namely:

1. Devising a valid solution strategy, which is to see if the solver applies the right concepts and procedures to produce a valid approach to solving the problem.

2. Mathematizing the word problem, i.e., choosing a specific strategy and using representations to characterize mathematical relationships in a given problem (e.g., algebraically, non-algebraically).
3. Executing a strategy to solve it correctly, using the chosen strategy to solve the problem so as to get the correct solution.

Furthermore, (Saygili, 2017), explains to see the level of student problem-solving seen from three aspects, namely conceptual understanding, procedural information, and problem-solving skills and strategies.

(J.M.Suh, 2014) provides ways to help students develop problem-solving strategies, namely as follows:

1. The teacher chooses a task that is rich in answers or solutions to a problem.
2. Involve students actively in math discussions.
3. Students model mathematics by visualizing problem situations.
4. Use appropriate tools to represent their thoughts.
5. Use mathematical structures.

b. Strategic flexibility

Students with high mathematical ability use a variety of strategies well and are adapted to the characteristics of the problem, while students with less mathematical ability rarely switch from the wrong initial strategy to the correct alternative (Fazio, 2016). This is a form of strategic flexibility in problem-solving. Strategic flexibility is the ability to use several strategies and change strategies flexibly according to the problem (Liu *et al*., 2018). In line with that, (Gavaz *et al*., 2021) said that strategic flexibility is a more comprehensive skill that requires not only knowledge of strategies but also the ability to choose the best strategy and switch between them. So it can be concluded that strategic flexibility is the ability of a person to use several strategies to solve one problem and be able to switch to another strategy to get a solution.

Strategic flexibility can be divided into two types, according to (Elia *et al*., 2009), namely inter-task flexibility (applying different strategies for each problem) and intra-task flexibility (changing strategy in the same problem). Then (Arslan & Yazgan, 2015) explain four criteria for strategic flexibility in problem-solving, namely:

1. Selection and use of the most appropriate strategy,
2. Changing strategies when it does not work for the solution of a problem,
3. Using multiple strategies for the solution of a problem, and
4. Changing strategies between problems

(Xu, 2017) classifies strategic flexibility becomes potential flexibility, namely competence or knowledge of various strategies (standard and innovative) to solve mathematical problems and practical flexibility, i.e., the ability to apply innovative strategies to specific problems. Furthermore, the results of research by Keleş and Yazgan (2022) found three themes and indicators of strategic flexibility that are:

1. Strategy adaptation, with two indicators, namely strategic knowledge and choosing the right strategy.
2. Intra-task strategic flexibility, with four indicators, namely changing strategies when it does not work, solving problems with different strategies, use several strategies simultaneously to solve problems, and checking the correctness of solutions with different strategies.

3. Inter-task strategic flexibility, one indicator is changing strategies when facing different problems.

Segura & Ferrando (2023), established three levels of flexibility in solving problems, namely:

1. Non-flexible, if propose the same solution strategy in all tasks.
2. Moderately flexible, if propose two different valid solution strategies but switched solution strategy only to one problem.
3. Very flexible, if propose two or more different valid solution strategies and switched solution strategy in two or more problems.

To increase strategic flexibility in problem-solving teacher candidates, then (Friesen *et al*., 2022) designs learning using two types of cartoon sketches, namely short cartoons, each describing a different problem-solving strategy (for example, work backward) and more complex cartoons provide an opportunity to analyze how students use various strategies and/or struggles when solving non-routine problems.

Conclusion

Based on the results of the literature review from several articles that have been described previously, it can be concluded that the strategies used by pre-service mathematics teachers in solving mathematical problems are diverse and are classified into strategic cognitive, strategic processes, and strategic solutions, how the strategy of pre-service mathematics teachers in solving problems can be seen from the strategies used in each stage. First, look at the strategies used in formulating and categorizing problems into mathematical concepts. This falls into the category of strategic cognition. Second, looking at the strategies used in the calculation process, whether using direct calculations or using algebraic or non-algebraic representation, is included in the category strategic processes, and the last look at the solution or the final result obtained from the use of a strategy that has been previously chosen or in other words strategic solutions. The strategic flexibility can be seen from strategic knowledge and choosing the right strategy, intra-task strategic flexibility, and inter-task strategic flexibility. Then, strategic flexibility can be categorized into three levels, namely non-flexible, moderately flexible, and very flexible.

Acknowledgements

The authors would like to express their gratitude to the Higher Education Financing Center (Balai Pembiayaan Perguruan Tinggi) and the Indonesia Endowment Fund for Education (Lembaga Pengelola Dana Pendidikan) for supporting the publication of this article.

References

Almeida, R., & Bruno, A. (2014). Strategies of pre-service primary school teachers for solving addition problems with negative numbers. *International Journal of Mathematical Education in Science and Technology*, *December*, 37–41. https://doi.org/10.1080/0020739X.2013.877605

Arslan, C., & Yazgan, Y. (2015). Common and Flexible Use of Mathematical Non Routine Problem Solving Strategies. *American Journal of Educational Research*, *3*(12), 1519–1523. https://doi.org/10.12691/education-3-12-6

Avcu, S., & Avcu, R. (2010). Pre-service elementary mathematics teacher's use of strategies in mathematical problem solving. *Procedia - Social and Behavioral Sciences*, *9*, 1282–1286. https://doi.org/10.1016/j.sbspro.2010.12.321

Awofala, A. O. A. (2017). Assessing senior secondary school students' mathematical proficiency as related to gender and performance in mathematics in Nigeria. *International Journal of Research in Education and Science*, *3*(2), 488–502. https://doi.org/10.21890/ijres.327908

Celebioglu, B., Yazgan, Y., & Ezentaş, R. (2010). Usage of non-routine problem solving strategies at first grade level. *Procedia - Social and Behavioral Sciences*, *2*(2), 2968–2974. https://doi.org/10.1016/j.sbspro.2010.03.449

Clements, D. H., Dumas, D., Dong, Y., Banse, H. W., & Day-hess, C. A. (2019). Strategy Diversity in Early Mathematics Classrooms. *Contemporary Educational Psychology*, 101834. https://doi.org/10.1016/j.cedpsych.2019.101834

Copur-gencturk, Y. (2021). Strategic competence for multistep fraction word problems : an overlooked aspect of mathematical knowledge for teaching. *Educational Studies in Mathematics*, 49–70.

Egodawatte, G., & Stoilescu, D. (2015). Grade 11 students' interconnected use of conceptual knowledge, procedural skills, and strategic competence in algebra: a mixed method study of error analysis. *European Journal of Science and Mathematics Education*, *3*(3), 289–305. https://doi.org/10.30935/scimath/9438

Elia, I., van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM - International Journal on Mathematics Education*, *41*(5), 605–618. https://doi.org/10.1007/s11858-009-0184-6

Fazio, L. (2016). Strategy use and strategy choice in fraction magnitude comparison. *Journal of Experimental Psychology: Learning Memory and Cognition*, *42*(1), 1–16. https://doi.org/10.1037/xlm0000153

Friesen, M., Knox, A., Friesen, M., & Knox, A. (2022). *Pre-service teachers learn to analyse students ' problem-solving strategies with cartoon vignettes To cite this version : HAL Id : hal-03745715 Pre-service teachers learn to analyse students ' problem-solving strategies with cartoon vignettes*.

Gavaz, H. O., Yazgan, Y., & Arslan, Ç. (2021). Non-routine problem solving and strategy flexibility: A quasi-experimental study. *Journal of Pedagogical Research*, *5*(3), 40–54. https://doi.org/10.33902/jpr.2021370581

Jiang, C., & Chua, B. L. (2010). *Strategies for solving three fraction-related word problems on speed: a comparative study between chinese and singaporean students*. *1986*(January 2007), 73–96.

Keleş, T., & Yazgan, Y. (2021). Gifted eighth, ninth, tenth and eleventh graders' strategic flexibility in non-routine problem solving .,. *The Journal of Educational Research*, DOI: 10.1080/00220671.2021.1937913.

Keleş, T., & Yazgan, Y. (2022). Indicators of gifted students' strategic flexibility in non-routine problem solving. *International Journal of Mathematical Education in Science and Technology*, *August*. https://doi.org/10.1080/0020739X.2022.2105760

Kilpatrick, J., Swafford, J., & B Findell, R. (2001). Adding It Up: Helping Children Learn Mathematics. In *Social Sciences*.

KOCA, A. K., & GÜRBÜZ, R. (2021). Problem Solving Strategies Used By Gifted Secondary School Students To Solve Math Problems *. *Bolu Abant İzzet Baysal University Journal of Faculty of Education*, *21*, 348–359.

Liu, R. de, Wang, J., Star, J. R., Zhen, R., Jiang, R.-H., & Fu, X.-C. (2018). Turning potential flexibility into flexible performance: Moderating effect of self-efficacy and use of flexible cognition. *Frontiers in Psychology*, *9*(May), 1–10. https://doi.org/10.3389/fpsyg.2018.00646

Öçal, M. F., Şen, C., Güler, G., & Kar, T. (2019). The investigation of prospective mathematics teachers ' non-algebraic solution strategies for word problems. *International Journal of Mathematical Education in Science and Technology*, *5211*. https://doi.org/10.1080/0020739X.2019.1597936

Ostler, E. (2011). Teachiing Adaptive and Strategic Reasoning Through Formula Derivation: Beyond Formal Semiotics. *International Journal of Mathematics Science Education Technomathematics Research Foundation*, *4*(2), 16–26.

Polya, G. (1981). *Mathematical Discovery Polya (1981, Wiley)*. https://www.isinj.com/mt-usamo/Mathematical Discovery Polya (1981, Wiley).pdf

Rubenstein, L. D. V., Callan, G. L., Ridgley, L. M., & Henderson, A. (2019). Students' strategic planning and strategy use during creative problem solving: The importance of perspective-taking. *Thinking Skills and Creativity*, *34*(December 2018). https://doi.org/10.1016/j.tsc.2019.02.004

Saygılı, S. (2017). Examining The Problem Solving Skills and The Strategies Used by High School Students in Solving Non-routine Problems. *E-International Journal of Educational Research*, *8*(2), 91–114.

Segura, C., & Ferrando, I. (2023). Pre-service teachers' flexibility and performance in solving Fermi problems. *Educational Studies in Mathematics*, *113*(2), 207–227. https://doi.org/10.1007/s10649-023-10220-5

Suh, J., Leong, K. M., Freeman, P., Meints, K., Corcoran, M., & Wills, T. (2012). *Fostering strategic competence for teachers through modeling rational numbers problem tasks*.

Suh, J. M. (2014). *Developing Strategic Competence by Teaching Using the Common Core Mathematical Practices, Defining Strategic Competence and Teaching through Mathematical Practices*. *April*, 77–87.

Tambychik, T., & Meerah, T. S. M. (2010). Students' difficulties in mathematics problem-solving: What do they say? *Procedia - Social and Behavioral Sciences*, *8*, 142–151. https://doi.org/10.1016/j.sbspro.2010.12.020

Xu, L. (2017). Measures of potential flexibility and practical flexibility in equation solving. *Frontiers in Psychology*, *8*. https://doi.org/10.3389/fpsyg.2017.01368